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6. AUTHOR(S)

Michael Shearer

7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES)

North Carolina State University
Box 8205
Raleigh, NC 27695-8205

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13. ABSTRACT (Maximum 200 words)

This project concerns properties of wave propagation in partial differential equations that are nonlinear and of hyperbolic type. The research is driven largely by applications to granular materials (notably soils in the current grant period), and to the propagation of shock waves in nonlinear elastic materials. Results utilize a combination of mathematical analysis and numerical simulation.

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SYSTEMS OF NONLINEAR HYPERBOLIC PARTIAL DIFFERENTIAL EQUATIONS

FINAL PROGRESS REPORT

MICHAEL SHEARER

DECEMBER, 1997

U.S. ARMY RESEARCH OFFICE

GRANT NUMBER DAAH04-94-G-0043

NORTH CAROLINA STATE UNIVERSITY

A. STATEMENT OF THE PROBLEM STUDIED

This project concerns properties of wave propagation in partial differential equations that are nonlinear and of hyperbolic type. The research is driven largely by applications to granular materials (notably soils in the current grant period), and to the propagation of shock waves in nonlinear elastic materials. Also of interest is the issue of whether a continuum description of granular materials is adequate at length scales of a few hundred grain diameters. For this purpose, the problem of describing stress fluctuations was addressed using a discrete model employing springs and frictional sliders to describe elastoplastic effects in a layer of granular material undergoing shear.

B. SUMMARY OF RESEARCH RESULTS

This description of results from the project covers research in two areas:

1. Properties of granular materials.

2. Hyperbolic conservation laws.

1. GRANULAR MATERIALS

The research into properties of granular materials concerned: (i) Wave propagation in models of saturated soils, including new theoretical results on the equations of hypoplasticity; (ii) Formation of shear bands; (iii) Discrete models to describe stress fluctuations observed experimentally.

(i) Wave propagation.

In papers [6, 7], we apply results on 2×2 systems of fully nonlinear equations from [4, 5] to the full 5×5 system of pdes describing the propagation of plane waves [Osinov]. Here, we deal with granular materials whose constitutive law is specified by the theory of hypoplasticity. The main result is that an initial discontinuity is resolved uniquely into propagating shock waves by invoking a viscosity condition; without the condition there is a family of solutions depending on several parameters. In [11], similar ideas are used to analyze the propagation of a boundary disturbance into the interior of a region of undisturbed soil. The results of this paper have a bearing on the interpretation of the phenomenon of liquefaction as a dynamic process in which the propagating wave mobilizes the entire body of material, thus initiating catastrophic collapse of the granular skeleton of the material.

(ii) Shear bands.

Papers dealing with the formation of a shear band, the constitutive law, and the sensitivity to imperfections or nonuniformity, appear in references [8,9,10]. The main result here is that because of the onset of linear ill-posedness in the pdes, the departure from a uniform solution is $\mathcal{O}(1)$ in the presence of even a small perturbation in the material properties. In particular, as the shear band forms, the velocity profile typically develops a discontinuity, whereas the uniform solution maintains a continuous (and linear) velocity profile up until shear band formation, which occurs everywhere simultaneously. Paper [10] focuses on verifying the theoretical predictions numerically, and on simulating more realistic models to test the applicability of the analytic conclusions.

Additionally, the paper [1] appeared during this grant period. It deals with the deformation following the appearance of a shear band, and includes numerical simulations in one dimension and time showing the propagation of elastic precursor waves and elastoplastic unloading waves, as predicted by the theoretical analysis.

(iii) Discrete Models.

In [12] a discrete spring-slider model for stress fluctuations in strain-hardening elastoplasticity is formulated and analyzed. The model has similarities to the Prager-Mroz model [Prager], except that the friction law is position dependent. The model was motivated by experiments measuring stress fluctuations [Howell, Miller]. Statistical considerations are introduced through the initial spacing of the grains, and elastoplasticity emerges as the average reponse of the system. Fluctuations are estimated by calculating the variance of the response; they show an interesting dependence on a dimensionless parameter that is associated with the roughness of the grains. A somewhat more sophisticated model is analyzed in [13], in which the spacing of grains is modeled by a Poisson process. The conference proceedings paper [15] summarizes the results from the discrete models. Numerical simulations using a discrete element model of particle interactions are carried out in [16].

2. HYPERBOLIC CONSERVATION LAWS

In [3], the Riemann problem is solved for a scalar conservation law with dissipation and dispersion, using undercompressive shock waves calculated explicitly. The solutions differ from those in the classical theory of conservation laws in that the new solutions include the phenomenon of wave splitting, and have the feature that the solutions need not be monotonic in space. Numerical results demonstrate that the new kinds of solution occur stably. Recently, [17] we showed that similar phenomena occur in more general scalar equations with nonconvex flux functions. In [17], a detailed analysis and numerical results are given for the Buckley-Leverett equation of two–phase flow in a porous medium in which dispersive effects are important, motivated by the numerical dispersion present in the truncation error of certain numerical schemes.

A more complicated analysis was performed earlier [2] for a nonlinear wave equation that loses hyperbolicity, This was followed up recently [18] by an analysis of the strictly hyperbolic quasilinear wave equation describing one–dimensional nonlinear elasticity with a stress–strain law that is a monotonic cubic, mimicing the constitutive law for rubber. The solutions involve two wave families, but each family contains all the phenomena of the scalar equation. However, undercompressive shocks only appear when the dispersion is sufficiently strong in comparison with dissipation, a result that differs from the scalar case.

Additionally, the paper [14] concerning the medical procedure of lithotripsy appeared in this grant period. It outlines issues concerning the behavior of spherical bubbles that are excited during the bombardment of kidney stones by shock waves. The new contribution of the paper is an asymptotic analysis of the equations describing the cycle of growth and collapse of a spherical bubble.

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D. PERSONNEL SUPPORTED

Michael Shearer; Matthew R. Schulze (MS, May 1996).

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